
RATIOS AND PROPORTIONS

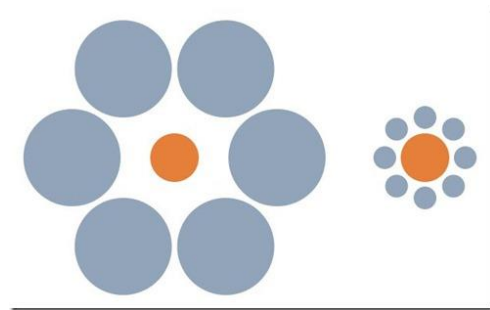
There are two major ways to compare two things. If Joseph is 40 years old and Ellie is 10 years old, we can say either:

- 1) Joseph is 30 years *older* than Ellie, or
- 2) Joseph is 4 *times as old* as Ellie.

The first way is found by
subtracting: $40 - 10 = 30$.

The second way is found by
dividing: $\frac{40}{10} = 4$

It's this second way, using division, that is very powerful in explaining many of the numerical relationships we see in the world. When we compare two things by *division*, we're creating a **ratio**.



❑ PROPORTIONS

Since a ratio is calculated by division, a ratio is basically a fraction. A **proportion**, however, is a statement that two ratios are equal. An example of a proportion is

$$\frac{16}{12} = \frac{8}{6}$$

Each fraction is a ratio.

These ratios are equal because both fractions could be reduced to $\frac{4}{3}$, or if you prefer, they both equal 1.333....

□ **SOLVING SIMPLE PROPORTIONS**

Example 1: Solve the proportion; that is, solve for x :

$$\frac{5}{13} = \frac{x}{1287}$$

We can use basic algebra to isolate the x by multiplying each side of the equation by 1287. It might be useful to write 1287 as a fraction with denominator 1:

$$\frac{5}{13} \left[\frac{1287}{1} \right] = \frac{x}{1287} \left[\frac{1287}{1} \right]$$

This simplifies to

$$\frac{6435}{13} = x, \text{ or } \underline{\underline{x = 495}}$$

Example 2: To solve the proportion

$$\frac{18}{n} = \frac{30}{17}$$

we notice that the variable n is on the bottom of the first ratio — we have to get it to the top so that we can solve for it. So first we'll multiply each side of the equation by n :

$$\frac{18}{n} [n] = \frac{30}{17} [n]$$

As before, we'll write each of the n 's that we've put into the problem as a fraction with denominator 1. Thus, we can write

$$\frac{18}{\cancel{n}} \left[\frac{\cancel{n}}{1} \right] = \frac{30}{17} \left[\frac{n}{1} \right]$$

which simplifies to

$$18 = \frac{30n}{17}$$

Now multiply each side of the equation by 17 (or $\frac{17}{1}$ if you'd prefer):

$$18[17] = \frac{30n}{17}[17],$$

which (after flipping around the equal sign) leads to the equation

$$30n = 306$$

Now divide each side by 30:

$$\frac{30n}{30} = \frac{306}{30}$$

and we have our solution: **$n = 10.2$**

Homework

- Which of the following statements are true proportions?
[Try reducing fractions or converting fractions to decimals.]

a. $\frac{60}{120} = \frac{1}{2}$	b. $\frac{37}{111} = \frac{10}{30}$	c. $\frac{4}{5} = \frac{5}{6}$
d. $\frac{1}{7} = \frac{2}{13}$	e. $\frac{336}{84} = \frac{68}{17}$	f. $\frac{14}{6} = \frac{7}{2}$

- Solve each proportion (leave answers rounded to 2 digits):

a. $\frac{x}{7} = \frac{2}{9}$	b. $\frac{14}{a} = \frac{1}{20}$	c. $\frac{2}{7.2} = \frac{b}{5}$
d. $\frac{100}{9.1} = \frac{2.3}{u}$	e. $\frac{c}{2.3} = \frac{1}{1.1}$	f. $\frac{14}{y} = \frac{14}{37}$

- Solve each proportion (leave answers rounded to 2 digits):

a. $\frac{a}{13} = \frac{14}{156}$

b. $\frac{23}{b} = \frac{1}{14}$

c. $\frac{3}{4} = \frac{n}{128}$

d. $\frac{9}{2} = \frac{7.8}{z}$

e. $\frac{13}{u} = \frac{23}{10}$

f. $\frac{17}{15} = \frac{0.23}{k}$

g. $\frac{x}{5} = -7$

h. $\frac{17}{n} = -\frac{4}{5}$

i. $\frac{a}{9} = \frac{9}{a}$ [2 answers]

j. $\frac{x}{3} = \frac{1}{7}$

k. $\frac{2.1}{y} = \frac{5}{7}$

l. $\frac{8}{9} = \frac{w}{0.8}$

m. $\frac{2.1}{5} = \frac{9.3}{z}$

n. $\frac{5.6}{a} = \frac{2.33}{0.04}$

o. $\frac{x}{2} = \frac{18}{x}$ [2 answers]

□ SOLVING MORE PROPORTIONS

I'm now going to show you a "shortcut" to solving proportions. Take the proportion:

$$\frac{a}{b} = \frac{c}{d}$$

Multiply both sides of the equation by the LCD, which is bd :

$$\frac{a}{b}[bd] = \frac{c}{d}[bd]$$

Now a little canceling,

$$\frac{a}{\cancel{b}}[\cancel{b}d] = \frac{c}{\cancel{d}}[b\cancel{d}]$$

which simplifies to

$$ad = bc$$

In short, we can take the proportion

$$\frac{a}{b} = \frac{c}{d},$$

"cross-multiply," and end up with the equivalent equation

$$ad = bc$$

Example 3: Solve for x : $\frac{3x-9}{5} = \frac{2x+7}{4}$

This proportion looks more complicated than the previous ones, but you know what? We can use our new "cross-canceling" shortcut to solve it. Let's begin by writing the original problem:

$$\frac{3x-9}{5} = \frac{2x+7}{4} \quad \text{(the original equation)}$$

$$4(3x-9) = 5(2x+7) \quad \text{(cross-multiplying)}$$

$$12x - 36 = 10x + 35 \quad \text{(distributing)}$$

$$2x - 36 = 35 \quad \text{(subtract } 10x \text{ from each side)}$$

$$2x = 71 \quad \text{(add 36 to each side)}$$

$$x = \frac{71}{2} \quad \text{(divide each side by 2)}$$

Homework

4. Solve each proportion (leave your answer in fractional form):

a. $\frac{x-1}{5} = \frac{2x+3}{4}$

b. $\frac{n}{3} = \frac{5-n}{10}$

c. $\frac{3a+9}{10} = \frac{2a}{3}$

d. $\frac{3}{m-2} = \frac{5}{m+12}$

e. $\frac{-2}{u-7} = \frac{-6}{9-u}$

f. $\frac{7}{-3y+7} = \frac{-1}{2-3y}$

g. $\frac{w+5}{w-3} = \frac{7}{8}$

h. $\frac{2a-1}{3} = \frac{2-a}{5}$

i. $\frac{2}{3} = \frac{4c-1}{4c+1}$

j. $\frac{-5}{2x+1} = \frac{3}{2-3x}$

k. $\frac{2y+1}{3y-4} = 8$

l. $\frac{x+3}{x-5} = \frac{x+1}{x-1}$

Solutions

1. a, b, and e are true

2. a. $x = 1.56$ b. $a = 280$ c. $b = 1.39$ d. $u = 0.21$
e. $c = 2.09$ f. $y = 37$

3. a. $a = 1.17$ b. $b = 322$ c. $n = 96$ d. $z = 1.73$
e. $u = 5.65$ f. $k = 0.2$ g. $x = -35$ h. $n = -21.25$
i. $a = \pm 9$ j. $x = 0.43$ k. $y = 2.94$ l. $w = 0.71$
m. $z = 22.14$ n. $a = 0.1$ o. $x = \pm 6$

4. a. $x = -\frac{19}{6}$ b. $n = \frac{15}{13}$ c. $a = \frac{27}{11}$
d. $m = 23$ e. $u = \frac{15}{2}$ f. $y = \frac{7}{8}$
g. $w = -61$ h. $a = \frac{11}{13}$ i. $c = \frac{5}{4}$
j. $x = \frac{13}{9}$ k. $y = \frac{3}{2}$ l. $x = -\frac{1}{3}$

“Only the curious will learn and only the resolute will overcome the obstacles to learning. The *quest quotient* has always excited me more than the intelligence quotient.”

Edmund Wilson